

# COMPUTER AIDED MODELLING OF A MULTIDIELECTRIC STRUCTURE AND ITS APPLICATION TO THE DESIGN OF OVERLAY COUPLERS

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## ABSTRACT

A computer model of a multi-dielectric multi-conductor system has been developed. The solution of the field equations of the system has yielded a powerful computer aided design tool suitable for use in the design of microstrip based overlay couplers. The method of analysis of the system is presented together with theoretical predictions of the performance of overlay couplers. Encouraging comparisons have been made between a set of previously measured results and the theoretical performance predicted using this method.

## INTRODUCTION

Theoretical models and design equations for microwave couplers have been available for some time. These models however, have generally been concerned with highly symmetrical planar or coplanar structures and are not therefore applicable to a multi-dielectric system in which the degree of symmetry is significantly reduced.

This submission presents a model of a system, suitable for use in overlay coupler design, in which up to three dielectric materials are used. Details of the method of solution of the associated field equations are presented. Software generated employing these solutions has been used to produce a theoretical design of an overlay coupler, and a comparison has been made with a coupler previously measured elsewhere.

Couplers designed in this way are currently being manufactured and evaluated at Marconi Electronic Devices Ltd.

## THE STRUCTURE TO BE ANALYSED

Typical cross sections of overlay couplers are shown in Figs.1 and 2. The enclosure is assumed to be grounded and dimensions A and B are large compared with W1 and W2. Poisson's equation relates the potential distribution to the charge density as below, subject to the boundary conditions given in Fig. 3.

$$\nabla^2 \Phi(x,y) = -\frac{1}{\epsilon} \rho(x,y). \quad \text{EQN (1)}$$

Green's function  $G(x,y/x_0,y_0)$  is defined as the potential at  $(x,y)$  due to a unit charge in an infinitely small volume at  $(x_0, y_0)$ , and solutions can be found by solving EQN 1 subject to boundary conditions similar to those in Fig. 3.

These solutions have been presented elsewhere [1] for the case where the charge distribution is assumed to be on an infinitely thin conductor on  $y = H1 + H2$ . Solutions for a charge distribution on  $y = H1$  are also required and these can be found in a similar way.

## DERIVATION OF A CAPACITANCE MODEL FOR THE SYSTEM:

In order to calculate the self and mutual capacitances for the two conductors each strip is considered as being made up of a number of substrips. These are numbered 1 to N for strip 1 and N+1 to 2N for strip 2. [Fig. 4]. Note that for two conductors of different widths, either different numbers of substrips or different substrip widths can be specified for each strip. For the case of 2N substrips, the potential on the  $j^{\text{th}}$  substrip is given by:

$$V_j = \sum_{i=1}^{2N} G_{ji} Q_i$$

where  $Q_i$  = charge on  $i^{\text{th}}$  substrip

$G_{ji}$  = Green's function for the potential on substrip j due to a unit charge on strip i.

If each potential  $V_j$  is included as an element of the matrix  $[V]_{2N}$  and similarly for  $Q_i$  and  $G_{ji}$  then:

$$[V]_{2N} = [G]_{2N,2N} [Q]_{2N}$$

$$\text{hence } [G]^{-1} [V] = [Q]$$

The matrix  $[G]^{-1}$  is therefore the capacitance matrix associated with the structure. Summing the capacitances of each substrip by summing the four quadrants of the matrix  $[G]^{-1}$  yields an expression for the total self and mutual capacitances of each strip. If  $Q_{T1}$  and  $Q_{T2}$  are the total charges on each conductor and  $V_{T1}$  and  $V_{T2}$  are the corresponding voltages, then:

$$\begin{bmatrix} Q_{T1} \\ Q_{T2} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} \begin{bmatrix} V_{T1} \\ V_{T2} \end{bmatrix}$$

$$\text{where } C_{11} = \sum_{i=1, j=1}^{N, N} \epsilon_{ij}$$

$$C_{12} = \sum_{i=1, j=N+1}^{N, 2N} \epsilon_{ig} \text{ etc.}$$

$$\text{and } [g] = [G]^{-1}$$

The above capacitance matrix can be used to calculate the modes of propagation of the system.

#### MODES OF PROPAGATION

It is important to note that due to the reduced symmetry of the structure, the usually assumed solution using odd and even modes of propagation does not exist. Instead the modes of propagation must be calculated using the coupling equations given in Fig. 5. These equations can be expressed in the form given below

$$\frac{d^2}{dx^2} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [C] [L] \frac{d^2}{dt^2} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

and

$$\frac{d^2}{dx^2} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = [L] [C] \frac{d^2}{dt^2} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

In a dispersionless system it is sufficiently accurate to assume that the inductance matrix  $[L]$  is equal to the inverse of the capacitance matrix when the latter is calculated with all dielectric constants set to unity. This assumption is often made in the calculation of the properties of microstrip lines and any errors introduced are found to be small.

The above equations [2] result in the eigenvalue, eigenvector problem given below:

$$\begin{aligned} [L] [C] - \begin{bmatrix} \lambda_n & 0 \\ 0 & \lambda_n \end{bmatrix} \frac{d^2}{dt^2} \begin{bmatrix} V_{1n} \\ V_{2n} \end{bmatrix} &= 0 \\ [C] [L] - \begin{bmatrix} \lambda_n & 0 \\ 0 & \lambda_n \end{bmatrix} \frac{d^2}{dt^2} \begin{bmatrix} I_{1n} \\ I_{2n} \end{bmatrix} &= 0 \end{aligned}$$

From the above expressions, two values of  $\lambda$  are found ( $\lambda_A$ ,  $\lambda_B$ ) and the corresponding two pairs of eigenvectors for voltage and current can then be calculated. These eigenvectors describe the two modes of propagation possible for the voltages and currents, and  $\lambda_A$  and  $\lambda_B$  give the relative effective dielectric constants for each mode.

#### DERIVATION OF Z AND S PARAMETERS FOR A FOUR-PORT

The voltages and currents on each strip can now be calculated. Each voltage and current can be expressed as a general sum of each of the four waves propagating in a strip. The four waves on each strip comprise two waves in each mode of propagation, travelling in opposite directions along the strip.

$$\begin{aligned} v &= A_1 \exp(-\lambda_A x) + A_2 \exp(\lambda_A x) \\ &+ A_3 \exp(-\lambda_B x) + A_4 \exp(\lambda_B x) \end{aligned}$$

By expressing both voltages and currents in this way and eliminating the constant terms  $A_1 - A_4$ , expressions for the port voltages shown in Fig. 6 can be derived in terms of the port currents. Hence the Z-matrix for the system can be found and this has been done elsewhere. [Ref 2].

By choosing a port termination of 50 ohms, the S-parameter matrix can be found in terms of the Z-parameters by simple matrix manipulation.

#### OVERLAY COUPLER DESIGN

An overlay coupler is required with 3dB coupling and an octave bandwidth. Software generated using the above solutions has been used to design single octave couplers for 2 different physical structures. Schematics for the two couplers are those given in Figs. 1,2. Theoretical responses are shown in Figs. 7 and 8 respectively.

The first of these designs has been configured so that, with repositioning of the two strips, higher coupling can be achieved. This is particularly useful in the design of multi-section, broader bandwidth couplers.

The second analysis is of a coupler, measured by H.R. Malone [Ref. 3] the design of which was based on a much simplified model. The theoretical results for the

Malone coupler design, analysed using this method, show a coupling factor almost exactly the same as Malone measured, however the measured results show a reduction in bandwidth. Malone observed that the inputs to his coupler appeared inductive (due to the method of launching onto the strips) and this was compensated for by adding capacitive stubs. These additional stray elements will produce a band limiting characteristic and this is shown in his results.

## CONCLUSIONS

A comprehensive solution to the field equations of a multi-dielectric, multi-conductor structure has been presented. These solutions have been incorporated into a computer aided design package which calculates the S-parameters of this general type of structure. Theoretical results show that couplers of this type should perform well over octave bandwidths and be compact and relatively simple to manufacture. Extensions to the design to give a 3-section broader bandwidth coupler are also possible. A comparison between the theoretical results calculated in this way and the results presented by Malone [Ref.3] show exceptionally close agreement.

## ACKNOWLEDGEMENTS

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## REFERENCES

- [1] Eikichi Yamashita and Kazuhiko Atsuki "Stripline with Rectangular Outer Conductor and three dielectric layers" MTT-18 May 1970 p238.
- [2] Vijai K. Tripathi. "Asymmetric Coupled Transmission lines in an inhomogeneous medium" MTT-23 Sept. 1975 p734.
- [3] H.R. Malone. "Microstrip overlay coupler suits broadband use" Microwaves & RF July 1985.

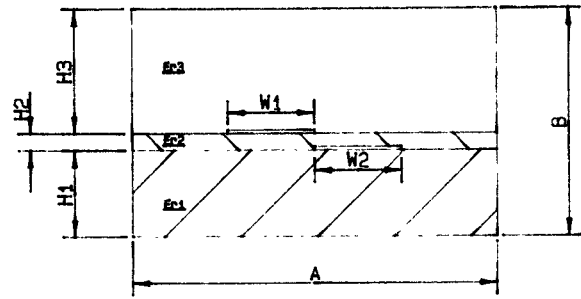


FIG 1: GENERAL STRUCTURE OF AN OVERLAY COUPLER

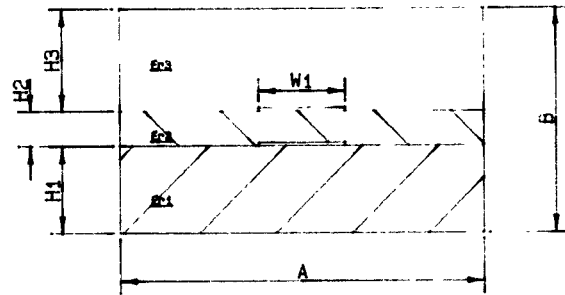


FIG 2: OVERLAY COUPLER ARRANGED FOR MAXIMUM COUPLING

FIG.3: BOUNDARY AND CONTINUITY CONDITIONS

Boundary Conditions

$$\begin{aligned}\Phi(0,y) &= 0 & \Phi(A,y) &= 0 \\ \Phi(x,0) &= 0 & \Phi(x,B) &= 0\end{aligned}$$

Continuity Conditions

$$\frac{\partial}{\partial x} \Phi(x, H_1 - 0) = \frac{\partial}{\partial x} \Phi(x, H_1 + 0)$$

$$\frac{\partial}{\partial x} \Phi(x, H_1 + H_2 - 0) =$$

$$\frac{\partial}{\partial x} \Phi(x, H_1 + H_2 + 0)$$

$$\epsilon_1 \frac{\partial}{\partial y} \Phi(x, H_1 - 0) = \epsilon_2 \frac{\partial}{\partial y} \Phi(x, H_1 + 0)$$

$$\epsilon_2 \frac{\partial}{\partial y} \Phi(x, H_1 + H_2 - 0) =$$

$$\epsilon_3 \frac{\partial}{\partial y} \Phi(x, H_1 + H_2 + 0) - p(x, H_1 + H_2)$$

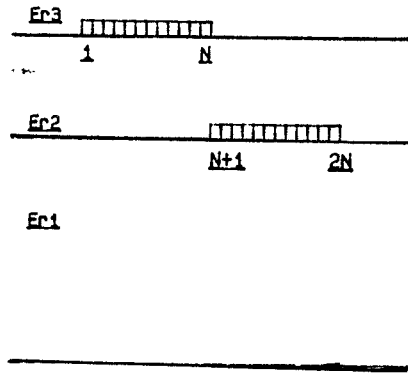


FIG 4: DIVISION OF CONDUCTORS INTO SUBSTRIPS

FIG. 5 COUPLING EQUATIONS

$$\frac{dV_1}{dx} = -L_{11} \frac{dI_1}{dt} - L_{12} \frac{dI_2}{dt}$$

$$\frac{dV_2}{dx} = -L_{12} \frac{dI_1}{dt} - L_{22} \frac{dI_2}{dt}$$

$$\frac{dI_1}{dx} = -C_{11} \frac{dV_1}{dt} - C_{12} \frac{dV_2}{dt}$$

$$\frac{dI_2}{dx} = -C_{12} \frac{dV_1}{dt} - C_{22} \frac{dV_2}{dt}$$

Where  $C_{i,j}$  and  $L_{i,j}$  are capacitances and inductances per unit length respectively.

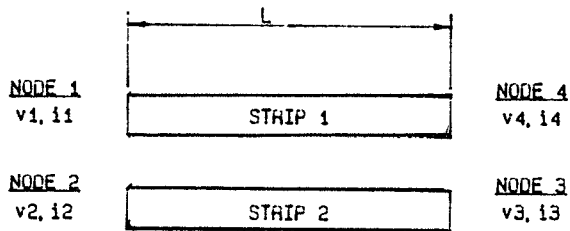


FIG 6: NODAL VOLTAGE AND CURRENT CONFIGURATION

FIGURE 7

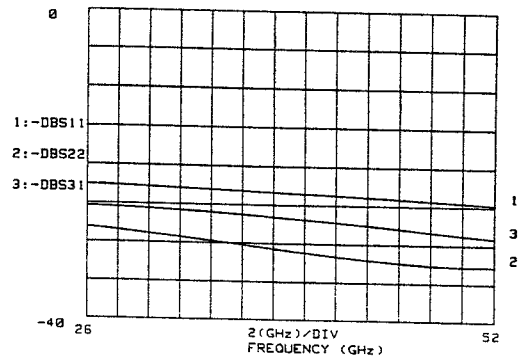
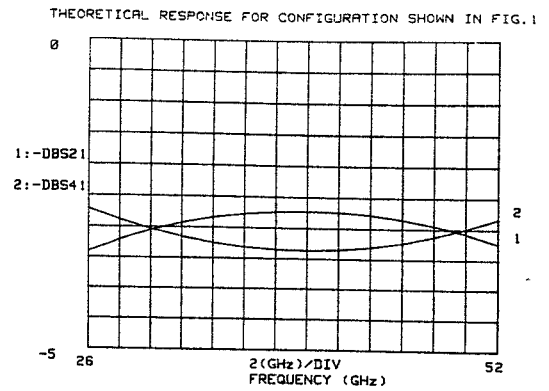


FIGURE 8

